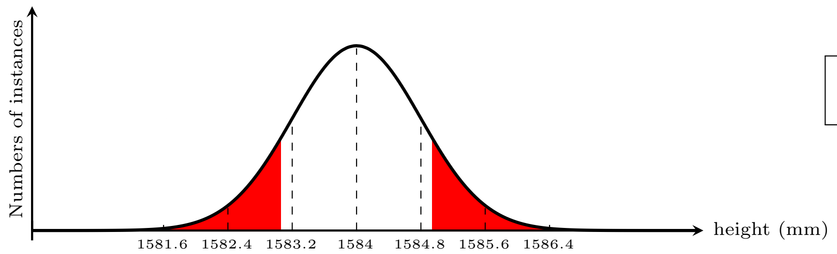
EXAMPLE: A furniture manufacturer gets a massive contract with a global retail company to produce 10,000 chairs of a specific design, to be sold in various different branches internationally. The manufacturer manages to make 10,000 chairs and is ready to ship them out. The heights of the chairs are normally distributed with mean 1584mm and a standard deviation of 0.8mm. All chairs *over* 1585mm in height and *under* 1583mm in height will be rejected, and not shipped. (i) What percentage of chairs are rejected? (ii) Suggest a way to reduce the rejection rate.

(i) First thing to do is draw a sketch, so that you can see what's going on. (Values for $\pm\sigma$, $\pm 2\sigma$, $\pm 3\sigma$ obtained by keeping adding and subtracting the given *'standard deviation'*, 0.8mm, from the given *'mean'* 1584mm.) We mark in shaded regions that indicate the rejected percentages.



$$z = \frac{x - \mu}{\sigma}$$

We then calculate our *'z-scores'* for our reject values:

$$z = \frac{1585\text{mm} - 1584\text{mm}}{0.8\text{mm}} = \frac{1\text{mm}}{0.8\text{mm}} = 1.25 \qquad z = \frac{1583\text{mm} - 1584\text{mm}}{0.8\text{mm}} = \frac{-1\text{mm}}{0.8\text{mm}} = -1.25$$

The *'z-score'* obtained for the upper limit is easy to solve. We can find the percentage of data values that are occurring less than or equal to a *'z-score'* of 1.25. We just read from the table.

$$P(X \leq 1585\text{mm}) = P(z \leq 1.25) = 0.8944$$

The result we want, however, is to know how much data is *over* 1585mm (N.B. not over and including, it's just everything over 1585mm.) We use the fact that the total area under the *'bell-shaped curve'* is 1, so we find the shaded area on the right tail of the distribution as follows:

$$P(X > 1585\text{mm}) = P(z > 1.25) = 1 - P(z \leq 1.25) = 1 - 0.8944 = 0.1056 \qquad (10.56\%.)$$

The calculation for the negative *'z-score'*, representing the percentage of chairs rejected on the left tail of the distribution, is done by symmetry. We cannot tell what a negative *'z-score'* is from the tables, because the lowest *'z-score'* given is $z = 0$. We use the following hack:

$$P(X < 1583\text{mm}) = P(z < -1.25) = P(z > 1.25)$$

We can do that because the *'normal distribution'* is symmetrical.

$$\implies P(X < 1583\text{mm}) = P(z < -1.25) = P(z > 1.25) = 1 - P(z \leq 1.25) = 1 - 0.8944 = 0.1056$$

Again that's 10.56%. So, in total, we have 10.56% + 10.56% = 21.12% of the chairs rejected.

(ii) Reduce the *'standard deviation'*, e.g. by using more *precise* production machinery!