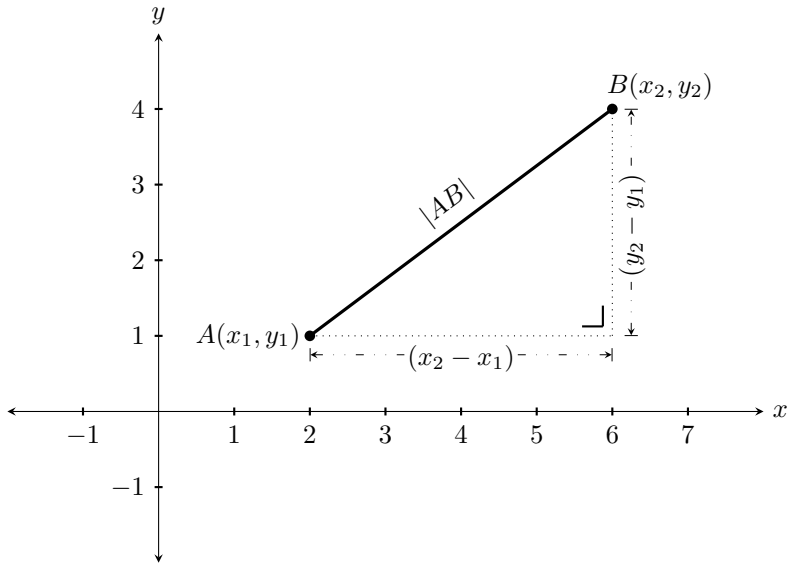
2.5 Distance Between Two Points Formula

The distance $|AB|$ between two coordinates $A(x_1, y_1)$ and $B(x_2, y_2)$, is given by the formula:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is basically the same as Pythagoras' theorem. Can you see why?



$$\boxed{|AB|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \ldots \qquad \textit{Pythagoras' theorem}$$

It doesn't matter which coordinates you assign to (x_1, y_1) or (x_2, y_2).

EXAMPLE: What is the distance between the points $(2, 1)$ and $(6, 4)$?

$(x_1, y_1) = (2, 1)$
$(x_2, y_2) = (6, 4)$

$$\begin{aligned} |AB| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6-2)^2 + (4-1)^2} \\ &= \sqrt{(4)^2 + (3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

$(x_1, y_1) = (6, 4)$
$(x_2, y_2) = (2, 1)$

$$\begin{aligned} |AB| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-6)^2 + (1-4)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

The two vertical bars either side of $|AB|$ mean the 'magnitude of' or the 'measure of' AB. That notation is used when we talk about the size of something. In this case, it's the size of the distance between two points. (See also the '*modulus*' in the '*Complex Numbers*' handout.)