EXAMPLE: (Arithmetic Series)
Prove, by induction, that the sum of n natural numbers is $\frac{n^2+n}{2}$, $n \in \mathbb{N}$.

Step 1: Prove that it works for $n = 1$.

$$\sum_{i=1}^{1} i = \frac{n^2+n}{2}$$
$$1 = \frac{(1)^2+(1)}{2}$$
$$1 = 1 \qquad \ldots \qquad \text{True}$$

Step 2: Assume true for all $n = k$, where $k \in \mathbb{N}$.

$$\sum_{i=1}^{k} i = \frac{k^2+k}{2} \qquad \ldots \qquad \textit{Induction Hypothesis}$$
$$\Rightarrow T_1 + T_2 + T_3 + \ldots + T_k = \frac{k^2+k}{2}$$
$$\Rightarrow 1+2+3+\ldots+k = \frac{k^2+k}{2} \qquad \ldots \qquad \boxed{T_k = 1+(k-1)1 = k}$$

Step 3: Test to see if it works for $n = k+1$, where k, $n \in \mathbb{N}$.

$$\sum_{i=1}^{k+1} i = \frac{(k+1)^2+(k+1)}{2}$$
$$\Rightarrow 1+2+3+\ldots+k+(k+1) = \frac{(k+1)^2+(k+1)}{2}$$
$$\Rightarrow \frac{k^2+k}{2} + (k+1) = \frac{(k+1)^2+(k+1)}{2} \qquad \ldots \quad \text{substituting ‘\textit{induction hypothesis}’}$$
$$\Rightarrow \frac{k^2+k}{2} + \frac{2(k+1)}{2} = \frac{(k+1)^2+(k+1)}{2} \qquad \ldots \qquad \textit{finding common denominator}$$
$$\Rightarrow \frac{k^2+k}{2} + \frac{2k+2)}{2} = \frac{(k+1)(k+1)+(k+1)}{2}$$
$$\Rightarrow \frac{k^2+k+2k+2}{2} = \frac{k^2+2k+1+(k+1)}{2}$$
$$\Rightarrow \frac{k^2+3k+2}{2} = \frac{k^2+3k+2}{2}$$

Step 4: Hence, by mathematical induction, since the sum of $k+1$ natural numbers is $\frac{(k+1)^2+(k+1)}{2}$, $k \in \mathbb{N}$, then the sum of n natural numbers is $\frac{n^2+n}{2}$, for all $n \in \mathbb{N}$.