

EXAMPLE: (Arithmetic Series)

Prove, by induction, that the sum of n natural numbers is $\frac{n^2+n}{2}$, $n \in \mathbb{N}$.

Step 1: Prove that it works for $n = 1$.

$$\begin{aligned} \sum_{i=1}^1 i &= \frac{n^2 + n}{2} \\ 1 &= \frac{(1)^2 + (1)}{2} \\ 1 &= 1 \quad \dots \quad \text{True} \end{aligned}$$

Step 2: Assume true for all $n = k$, where $k \in \mathbb{N}$.

$$\begin{aligned} \sum_{i=1}^k i &= \frac{k^2 + k}{2} \quad \dots \quad \text{Induction Hypothesis} \\ \Rightarrow T_1 + T_2 + T_3 + \dots + T_k &= \frac{k^2 + k}{2} \\ \Rightarrow 1 + 2 + 3 + \dots + k &= \frac{k^2 + k}{2} \quad \dots \quad \boxed{T_k = 1 + (k - 1)1 = k} \end{aligned}$$

Step 3: Test to see if it works for $n = k + 1$, where $k, n \in \mathbb{N}$.

$$\begin{aligned} \sum_{i=1}^{k+1} i &= \frac{(k+1)^2 + (k+1)}{2} \\ \Rightarrow 1 + 2 + 3 + \dots + k + (k+1) &= \frac{(k+1)^2 + (k+1)}{2} \\ \Rightarrow \frac{k^2 + k}{2} + (k+1) &= \frac{(k+1)^2 + (k+1)}{2} \quad \dots \quad \text{substituting 'induction hypothesis'} \\ \Rightarrow \frac{k^2 + k}{2} + \frac{2(k+1)}{2} &= \frac{(k+1)^2 + (k+1)}{2} \quad \dots \quad \text{finding common denominator} \\ \Rightarrow \frac{k^2 + k}{2} + \frac{2k + 2}{2} &= \frac{(k+1)(k+1) + (k+1)}{2} \\ \Rightarrow \frac{k^2 + k + 2k + 2}{2} &= \frac{k^2 + 2k + 1 + (k+1)}{2} \\ \Rightarrow \frac{k^2 + 3k + 2}{2} &= \frac{k^2 + 3k + 2}{2} \end{aligned}$$

Step 4: Hence, by mathematical induction, since the sum of $k + 1$ natural numbers is $\frac{(k+1)^2 + (k+1)}{2}$, $k \in \mathbb{N}$, then the sum of n natural numbers is $\frac{n^2+n}{2}$, for all $n \in \mathbb{N}$.

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