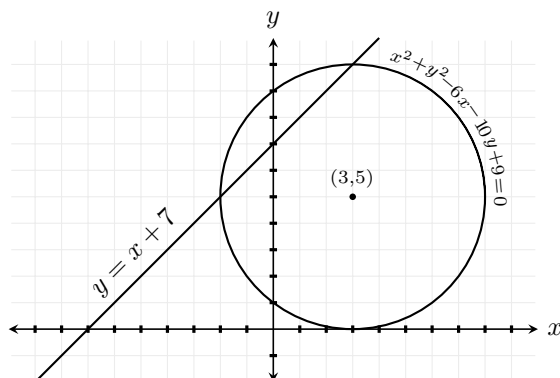


6.13 Finding The Point(s) Of Intersection Of A Circle And A Line

EXAMPLE:

Find the points of intersection of the circle $x^2 + y^2 - 6x - 10y + 9 = 0$ and the line $x - y = -7$.



Solve both the equation of the line and the equation of the circle simultaneously:

For the line: express one variable in terms of another, **then substitute** into equation of the circle.

$$x - y = -7 \quad \Rightarrow \quad y = x + 7$$

Substituting: $x^2 + (x + 7)^2 - 6x - 10(x + 7) + 9 = 0$... we end up with a quadratic,
 $\Rightarrow x^2 + (x + 7)(x + 7) - 6x - 10x - 70 + 9 = 0$ expressed in terms of x , alone.

$$\Rightarrow x^2 + x^2 + 7x + 7x + 49 - 6x - 10x - 70 + 9 = 0$$

$$\Rightarrow 2x^2 - 2x - 12 = 0$$

$$\Rightarrow x^2 - x - 6 = 0 \quad \dots \text{dividing both sides by 2}$$

$$\Rightarrow (x + 2)(x - 3) = 0$$

$$\Rightarrow x = -2 \quad \text{and} \quad x = 3$$

Substitute your obtained x coordinates into equation of **the line**, not the circle.^{vii}

$$y = x + 7$$

$$y = (-2) + 7 \\ = 5$$

$$y = (3) + 7 \\ = 10$$

Wrong:

$$x^2 + y^2 - 6x - 10y + 9 = 0$$

$$\begin{array}{l} (-2)^2 + y^2 - 6(-2) - 10y + 9 = 0 \\ 4 + y^2 + 12 - 10y + 9 = 0 \\ y^2 - 10y + 25 = 0 \\ (y - 5)(y - 5) = 0 \\ y = 5 \quad \text{and} \quad y = 5 \end{array} \quad \begin{array}{l} (3)^2 + y^2 - 6(3) - 10y + 9 = 0 \\ 9 + y^2 - 18 - 10y + 9 = 0 \\ y^2 - 10y = 0 \\ (y - 10)(y - 0) = 0 \\ y = 10 \quad \text{and} \quad y = 0 \end{array}$$

\therefore The points of intersection are $(-2, 5)$ and $(3, 10)$. (Compare with graph, above.)

^{vii}It will work for both the line and the circle equations, because both points are on the line and the circle, concurrently. When you plug a single coordinate into the equation of a circle, it can give you two answers, however. That's because you are effectively plugging in the equation of a horizontal or vertical line e.g. " $y=0$ ", " $x=-2$ " or " $x=3$ ", and there can be two places of intersection.

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